Analysis of Principal Component Analysis (PCA) 
Face Recognition: Effects of Similarity Measure

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**Abstract**— In this paper, the effects of similarity measure to the performance of PCA based face recognition are presented. A fundamental challenge in face recognition lies in determining which steps are important in the recognition of faces. Several studies have indicated the significance of certain steps in this regard, particularly pre-processing and feature extraction. Surprisingly, however, it has not been made clear whether the similarity measures play an important role in the recognition of faces. Twelve similarity measures have been used for the classification. Extensive experiments have been conducted on ORL and Yale face databases. The experimental results show the importance of using appropriate similarity measure.

**Keywords**— Security, Biometrics, Face Recognition, Principal Component Analysis, Eigenspace, Similarity Measure

I. INTRODUCTION

Face recognition has been studied extensively for more than 40 years. Now it is one of the most imperative sub-topics in the domain of face research \cite{[1]}-\cite{[4]}. Face recognition is a technology which recognize the human by his/her face image. Face recognition can be divided into two core approaches namely, content-based and appearance based \cite{[1]}.

Content-based recognition is based on the relationship between facial features like eyes, mouth & nose etc.

In appearance based recognition the face is treated as a two dimensional pattern of intensity variation. The face matching is done through its underlying statistical regularities.

Principal Component Analysis (PCA) has been proven to be an effective approach for the face recognition \cite{[5]}-\cite{[11]}. Sirovich and Kirby (1987 & 1990) used the eigenfaces for efficiently representing the face images using principal component analysis \cite{[12]}, \cite{[13]}. In 1991 Turk and Penland developed a face recognition system using PCA \cite{[7]}, \cite{[6]}. Then onwards the PCA has been widely used in face recognition and is considered as one of the most successful algorithm. It reduces the dimension effectively without losing the primary information.

Using PCA we find the eigenvectors (Principal Component or Principal Direction) in a set of training faces. Then we project faces in to this eigenvectors and get feature vectors. Matching is performed by calculating the similarity between these vectors. Usually comparison of face images is performed by Euclidean distance between feature vectors. Some times the angle based distance also used. Although there are many more other similarity measures, we were able to find only few attempts to compare and use other similarity measures in order to achieve better recognition.

This paper employed for finding effects of similarity measure in face recognition. In section II the PCA based face recognition system is discussed. The different similarity measures discussed in section III and experimental results are listed in section IV. Finally, sections V conclude and suggest the future scope.

II. PCA

Principal component analysis (PCA) was also called as Eigenface \cite{[6]}. The following steps summarize the process:

1. Let a face image $X(x, y)$ be a two dimensional mXn array (8-bit Gray Scale) of intensity values. An image may also be considering the vector of dimension mn, so that a typical image of size 112x92 becomes a vector of dimension 10304. Let the training set of images $\{X_1, X_2, X_3... X_N\}$. The average face of the set is defined by

   $$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$ (1)

2. Calculate the covariance matrix to represent the scatter degree of all feature vectors related to the average vector. The covariance matrix $C$ is defined by

   $$C = \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X})(X_i - \bar{X})^T$$ (2)

3. The Eigenvectors and corresponding eigenvalues are computed by using

   $$CV = \lambda V$$ (3)

   Where $V$ is the set of eigenvectors associated with its eigenvalue $\lambda$.

4. Sort the eigenvector according to their corresponding eigenvalues from high to low

5. Each of the mean centred image project into eigenspace using
6. In the testing phase each test image should be mean centred, now project the test image into the same eigenspace as defined during the training phase.

7. This projected image is now compared with projected training image in eigenspace. Images are compared with similarity measures. The training image that is closest to the test image will be matched and used to identify.

III. SIMILARITY MEASURES

Let \( x, y \) be the feature vectors of length \( n \). then we can calculate the following distances between these feature vectors [6]

A. EUCLIDEAN DISTANCE:
The Euclidean distance also called as L2 distance. \( L_2 \) is computed from the sum of square of the edge distances

\[
\delta(x, y) = \|x - y\|_2 = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}
\]  

B. CITY BLOCK DISTANCE:
This distance metric is variously named as block distance, \( L_1 \) distance or city block distance. The \( L_1 \) or block distance is calculated from summing the edge distances.

\[
\delta(x, y) = \|x - y\|_1 = \sum_{i=1}^{n} |x_i - y_i|
\]  

C. MINKOWSKI DISTANCE:
Minkowski distance (\( L_m \)) is computed from the sum of \( m^{th} \) power of the edge distance.

\[
\delta(x, y) = \|x - y\|_m = \left( \sum_{i=1}^{n} (x_i - y_i)^m \right)^{1/m}
\]  

for special case of \( m=1 \), the Minkowski distance gives the City Block distance, and for \( m=2 \), the Minkowski distance gives the Euclidean distance.

D. COSINE DISTANCE:
The Cosine distance is computed by

\[
\delta(x, y) = 1 - \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}}
\]  

E. CORRELATION DISTANCE:
The Correlation distance is computed by

\[
\delta(x, y) = 1 - \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}
\]  

Where \( \bar{x} = \sum_{i=1}^{n} x_i \) & \( \bar{y} = \sum_{i=1}^{n} y_i \)

F. BHATTACHARYYA DISTANCE:

\[
\delta(x, y) = \sum_{i=1}^{n} \sqrt{x_i y_i}
\]  

G. SQUARED EUCLIDEAN DISTANCE (SSE):

\[
\delta(x, y) = \|x - y\|_2^2 = \sum_{i=1}^{n} (x_i - y_i)^2
\]  

H. MEAN SQUAR DISTANCE (MSE):

\[
\delta(x, y) = \frac{1}{n} \|x - y\|_2^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - y_i)^2
\]  

I. CHI SQUAR DISTANCE (CSD):

\[
\delta(x, y) = \sum_{i=1}^{n} \frac{(x_i - y_i)^2}{x_i + y_i}
\]  

J. CANBERRA DISTANCE (CD):

\[
\delta(x, y) = \sum_{i=1}^{n} \left| \frac{x_i - y_i}{x_i + y_i} \right|
\]  

K. MODIFIED MANHATTAN DISTANCE (MMD):

\[
\delta(x, y) = \frac{\sum_{i=1}^{n} |x_i - y_i|}{\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i}
\]  

L. MODIFIED SSE-BASED DISTANCE (MSSE):

\[
\delta(x, y) = \frac{\sum_{i=1}^{n} (x_i - y_i)^2}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2}
\]  

IV. EXPERIMENTS

A. Databases

The Olivetti Research Lab (ORL) Database [14] of face images provided by the AT&T Laboratories from Cambridge University has been used for the experiment. It was collected between 1992 and 1994 [15]. It contains slight variations in illumination, facial expression (open/closed eyes, smiling/not smiling) and facial details (glasses/no glasses). It is of 400 images, corresponding to 40 subjects (namely, 10 images for each class). Each image has the size of 112 x 92 pixels with 256 gray levels. Some face images from the ORL database are as follows:

![Fig. 1. Some Face images from ORL Database](image-url)
The Yale Face database [16] [17] contains 11 frontal face images of 15 subjects, giving a total of 165 images. Each image has the size of 320 × 243 pixels with 256 gray levels. Lighting variations include left-light, center-light, and right-light. Spectacle variations include with-glasses and without-glasses. Facial expression variations include normal, happy, sad, sleepy, surprised, and wink. Some face images from the Yale face database are as follows:

B. Experimental Setup

The experiment has been done on two face databases ORL face database and Yale face database, with different number of training images i.e. five, six, seven, eight, nine, and ten. For testing all the images in the database has been considered. The PCA developed in MATLAB 7.0. In all the experiments the feature vectors has been calculated using PCA, only change the similarity measure between the feature vectors. For the comparison the Receiver Operating Characteristic (ROC)-based measures i.e. Equal Error Rate (EER) and Receiver Operating Characteristic Area (ROCA).

C. Results and Discussion

The experimental results are summarized in Tables I and II. Table I summarizes the results on the ORL face database and Table II summarizes the results on the Yale face database. From these tables we can how different similarity measures affect performance of the face recognition system. For measuring the overall goodness of the similarity measure with respect to the verification accuracy, we use area below the receiver operating characteristic curve.

The graphical representations of the results are shown in the figures 4-7. The smaller values of the EER and ROCA means better the results.

<table>
<thead>
<tr>
<th>Similarity Measures</th>
<th>5</th>
<th>6</th>
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<th>8</th>
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<td>168.4551</td>
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Fig. 3. Receiver Operating Characteristic.
### Table II: Recognition Results for Yale Database

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<td>2520.31</td>
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</table>

**Fig. 4.** Comparison of Similarity Measures with respect to equal error rate and Number of training images

**Fig. 5.** Comparison of Similarity Measures with respect to equal error rate and Number of training images

**Fig. 6.** Comparison of Similarity Measures with respect to equal error rate and Number of training images

**Fig. 7.** Comparison of Similarity Measures with respect to equal error rate and Number of training images
V. CONCLUSIONS

In this paper we compared twelve (12) similarity measures for the principal component analysis based face recognition. The experiments were performed on the ORL and Yale Face databases. The experiments show that the City block and Canberra are the first two best measures with respect to EER and ROCA of the face recognition system. We also showed that similarity measure has to be also considered as an important step in the face recognition system.

REFERENCES


http://www.uk.research.att.com/facedatabase.html

