Lossless Image Data Compression through LU Decomposition

Abstract:

Image data requires more space for storage, huge computational power and also requires more bandwidth while transmission in communication system. Each format of data contains lot of redundant information's. Hence it is necessary to compress the image data, there are many image compression techniques used to utilize storage and bandwidth in communication system, and these are basically spatial domain and frequency domain. Spatial domain is operates the image on gray scale values. Where as frequency domain transforms the signals and converts them into another domain called the transform coding, and these are further fall into two broad classes Lossless and Lossy techniques. Variable length coding, Huffman coding, Bit plane coding, Arithmetic Coding, LZW coding, Lossless Predictive Coding etc., are commonly used coding techniques for Lossless data compression and are normally provide the compression ratio of 2 to 10 and they are equally applicable to both binary to gray scale images [5]. A compression ratio of 1 to 20 can be obtained for artificial images with TIFF file format in LZW coding technique [3]. In this paper, we have used the Gauss elimination through LU decomposition techniques and applied on different images with TIFF file format. And, we are getting a compression ratio of more than 20.

Keywords: LU Decomposition, Lossless Image Compression, Data Compression

1. Introduction:

"A picture is a worth of thousand words". This is all the more true in the modern era (digital), which the information has become one of the most valued assets. A thousand words stored on digital computer requires very little capacity, but a single pictures can require a very large amount of storage while the advancement of computer storage technology continues rapid pace, a means reducing the storage requirements of an still image needed in most situations. Thus the Science of Digital image compression has emerged. Currently it is recognized as an "Enabling Technology". For example Image Compression has been continuous to be crucial to the growth of the multimedia computing. In addition it is the natural technology for handling the increased spatial resolutions of today's imaging sensors and evolving broadcast television standards. Further more, image compression plays important role in tele-video conferencing, remote sensing, document and medical imaging FAX, and the control of remotely piloted vehicles in military etc., [4]. Compression of data is very essential ingredient in any data storage, large transmission bandwidth and long transmission time for image, Audio and Video data [6]. By compression we mean removal of redundant information in any data. There are many types of compression algorithms are yet developed. These algorithms are fall into two broad classes 1. Lossless algorithms. 2. Lossy algorithms

1.1 Lossless compression:

In Lossless data compression the original data can be recovered exactly from the compressed data. It is generally used for application where cannot allow any difference between the compressed and the reconstructed data. Lossless compression techniques generally are composed of relatively two independent operations 1.An representation in which its Inter-pixel redundancies are reduced. 2. Coding the representation to eliminate coding redundancies. Variable length coding, Huffman coding, Arithmetic coding, LZW coding, Bit plane coding, Lossless Predictive coding are the most commonly used coding techniques for Lossless data compression and normally providing a compression ratio of 2 to 10 and they are equally applicable to both binary to gray scale images [5]. A compression ratio of 1 to 20 can be obtained for artificial images with TIFF file format in LZW coding technique [3].

1.2 Lossy compression:

Lossy compression techniques involve some loss of information and data cannot be recovered the same. These methods are used where the some loss of data can be acceptable. For. Example. A compressed Video signal is different from the original. However, we can generally obtain the higher compression ratio than the Lossless compression methods. Some of the common techniques for Lossy compression are Lossy predictive coding, Transform coding, Zonal coding, Wavelet coding, Image compression standard etc., [5]. Lossy compression techniques are much more effective at compression than the Lossless methods. The higher the compression ratio, the more noise added to the data [8].

2. Principles behind Compression:

A Common characteristic of most images is that the neighboring pixels are correlated and existing redundant information. Two fundamental components of compression are redundancy and irrelevancy reduction. Redundancy reduction aims to removing duplication from the signal source (Image and Video). Irrelevancy reduction omits parts of the signal that will not be noticed by the signal receiver, namely the Human Visual System (HVS). In general, three types of redundancy are identified.

-Spatial redundancy or Correlation between neighboring pixel values.

-Spectral redundancy or Correlation between different color planes or spectral bands.

-Temporal redundancy or Correlation between adjacent frames in sequence of images i.e. in Video applications.

Image compression research aims to reducing the numbers of bits needed to represent an image by removing the spatial and spectral redundancies as much as possible [6].

3. Image Compression Methodologies

Many methods have been presented over the past years to perform image compression having a one common goal: to alter the representation of information contained in an image. So that it can be represented sufficiently well with less information. Regard less of the details of each compression method, and these methods are fall into two broad categories 1. Lossless algorithms. 2. Lossy algorithms. Unfortunately, current Lossless algorithms provide relatively, small compression factors compared to Lossy methods.

3.1 LU Decomposition:

LU decomposition is one of the applications of Matrix factorization, and the primary goal of LU decomposition is that the time-consuming elimination step can be formulated and involves operations on the matrix of coefficients. LU decomposition method is used to decompose the image and evaluate in an efficient manner. Interestingly, Gauss elimination itself can be expressed as LU decomposition. For more mathematical overview of the LU decomposition strategy can be obtained in [9][2][7].

3.2 Overview of LU Decomposition:

Gauss elimination is designed to solve system of linear algebraic equations

$$[A]\{X\}=\{B\} \quad \ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

The equation (1) can be generalized for n-dimensional system and limited to three simultaneous equations and equation (1) can be rearranged to give

$$[A]\{X\}-\{B\}=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Equation (2) could be expressed as an upper triangular system

$$\begin{pmatrix} u11 & u12 & u13 \\ 0 & u22 & u23 \\ 0 & 0 & u33 \end{pmatrix} \begin{Bmatrix} x1 \\ x2 \\ x3 \end{Bmatrix} = \begin{Bmatrix} d1 \\ d2 \\ d3 \end{Bmatrix} \ldots\ldots (3)$$

Equation (3) can be expressed in matrix notation and rearranged to give

$$[U]\{X\}-\{D\}=0 \ldots\ldots\ldots\ldots\ldots. (4)$$

Now, Assume that there is a lower diagonal matrix with 1's on the diagonal,

$$[L]=\begin{pmatrix} 1 & 0 & 0 \\ l21 & 1 & 0 \\ l31 & l32 & 1 \end{pmatrix} \ldots\ldots\ldots\ldots\ldots. (5)$$

The property of equation (4) is permultiplied by it; equation (2) is the result, i.e.

$$[L]\{[U][X]\}-\{D\}\}=[A]\{X\}-B \ldots\ldots. (6)$$

if this equation holds, it follows from the rules for matrix multiplication that

$$[L][U]=[A] \ldots\ldots\ldots\ldots\ldots\ldots. 7)$$

and

$$[L][\{D\}=\{B\}\ldots\ldots\ldots\ldots\ldots.. (8)$$

A two step strategy see Figure 1 for obtaining solutions can be based on equation (4), (7) and (8):

1. LU decomposition step: [A] is factored or "decomposed" into lower [L] and upper [U] triangular matrices.
2. Substitution step: [L] and [U] are used to determine a solution {X} for a right hand side {B}. This step itself consist two steps. First equation (8) is used to generate an intermediate vector {D} by forward substitution. Then, the result is

substituted into equation (4), which can be solved by back substitution for {X} [7].

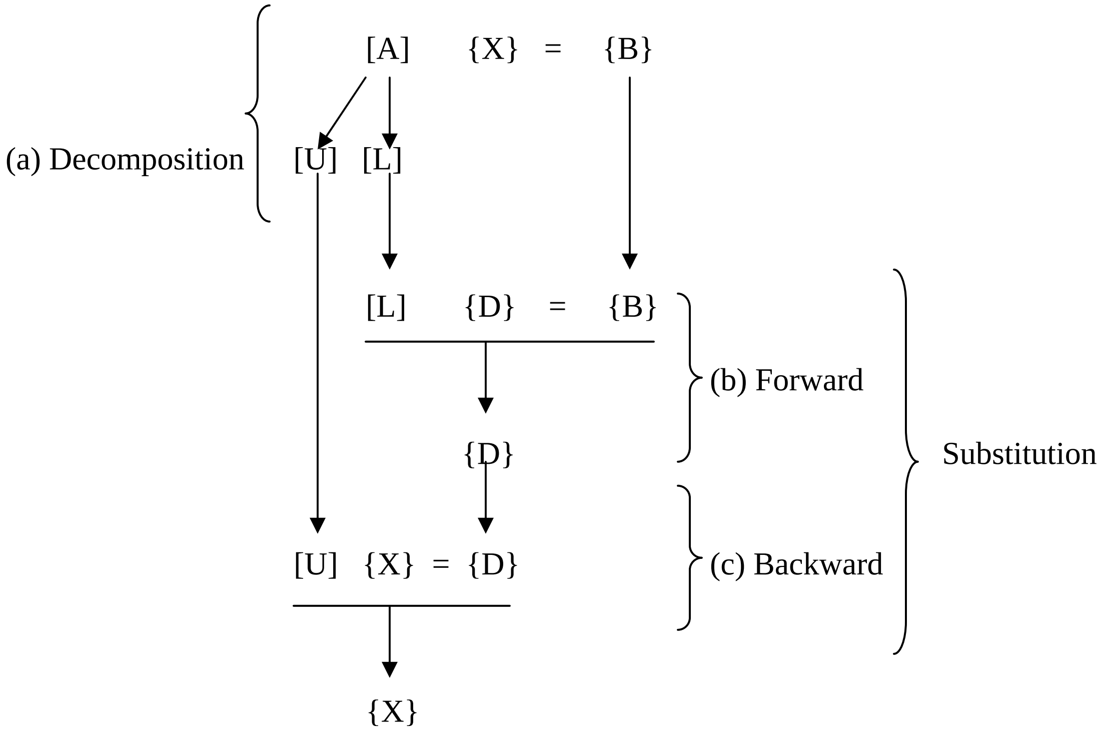

Figure 1. Steps in LU decomposition

4 Experimental Analyses

In this paper we have applied the LU decomposition method to different images with TIFF file format. This decomposition method decomposes the image into two parts L and U, represents the Lower and Upper elements of the given image respectively. The original image can be obtained by multiplying L*U, where the minor difference can be obtained due to round-off values. And we have calculated the Compression ratio and which is given in the following Table 1.

Image type	Size of the Original image in kb	Size of the Reconstructed image (L*U) in kb	Size of the Decomposed image (L+U) in kb	Compression Ratio in %
cameraman.tif	63.70	63.50	46.30	27.3155
sell.tif	30.30	30.20	20.47	32.4422
circuit.tif	74.90	74.80	45.70	38.9853
eight.tif	58.80	58.10	40.00	31.3894
forest.tif	121.00	120.00	71.90	40.5785
Kids.tif	92.90	91.04	85.00	08.5038
m83.tif	144.00	143.00	129.60	10.0000
canoe.tif	68.00	66.50	53.60	21.1765
pout.tif	67.30	67.30	46.51	30.8915
Moon.tif	179.00	179.00	144.50	19.2737

Table 1: Shows the Compression Ratio for Different images

5. Conclusion:

There are many techniques for Lossless image data compressions like Variable length coding, Huffman coding, Arithmetic coding, LZW coding, Bit plane coding, Lossless Predictive coding are the most commonly used coding techniques for Lossless data compression and normally providing a compression ratio of 2 to 10 and they are equally applicable to both binary to gray scale images [3]. A compression ratio of 1 to 20 can be obtained for artificial images with TIFF file format in LZW coding technique [2]. In this paper we have applied LU decomposition method for different images with TIFF file format and noted the size of U and L, and calculated the compression ratio, and we are getting the original image by multiplying L*U, where the minor difference can be obtained due to round-off values. Through this LU decomposition technique we have achieved compression ratio more than 20.

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